How do university students solve problems in vector calculus? Evidence from eye tracking

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Overall aims of the project:

To use eye-tracking to study students' problem solving strategies in higher education mathematics:

I To better understand and quantify key processes (the use of illustrations, symbols and formulas) that leads to successful problem solving.

II To develop knowledge for new computer based teaching and the optimization of teaching material.

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Outline of study:

Hypothesis/Questions: **”A figure always helps!” ?**

*** How do the illustrations affect comprehension?**

*** How do the illustrations affect the allocation of visual attention?**

Participants: **A total of 36 second year students from the engineering physics program (F) at Lund Institute of Technology (LTH), hence a fairly “uniform population”.**

Groups: **two groups with/without illustrations (20/16).**

Stimuli: **8 problems from vector calculus.**

Eye movements: **recorded at 250 Hz (with RED250)**

Recorded speech: **“think aloud” data for qualitative analyses.**

Example of a problem (P7) from the study:

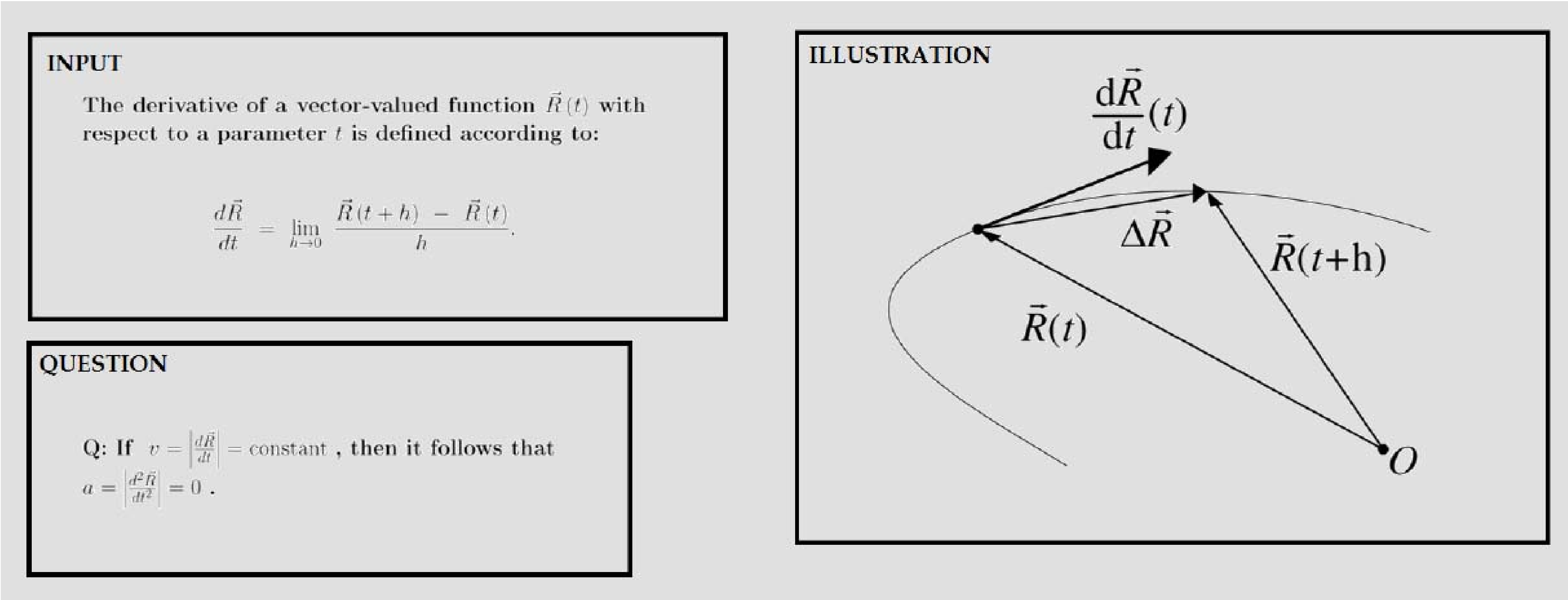
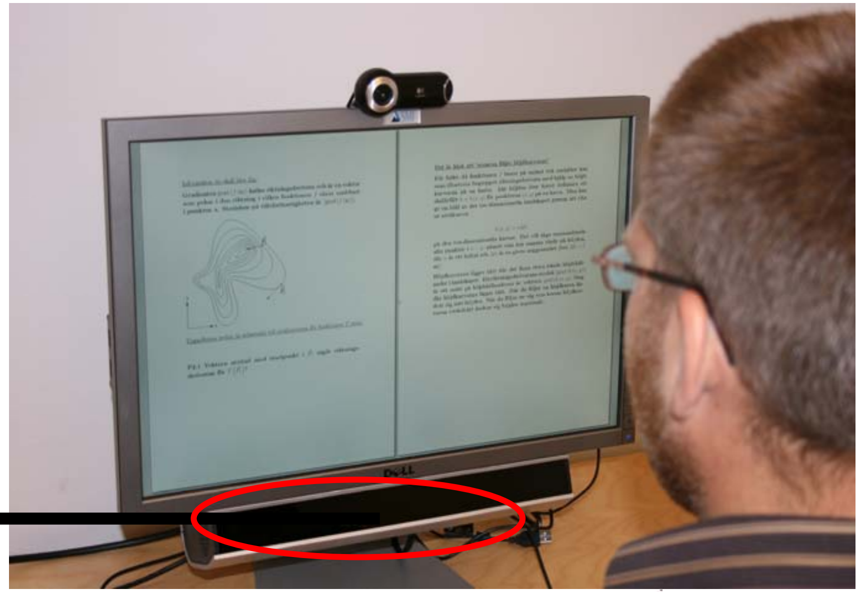
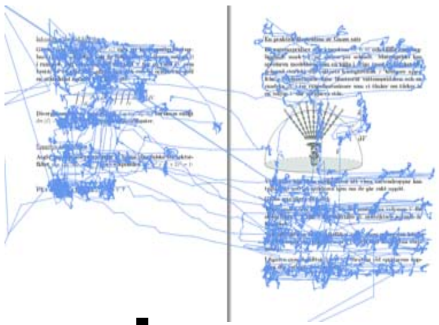


Figure 2: A stimulus with three overlayed areas of interest (AOIs): input, question, and illustration.

After 2 minutes the participants had to answer if Q is *true* or *false*, and to give their degree of *confidence* of the answer (1-7).

Experimental procedure

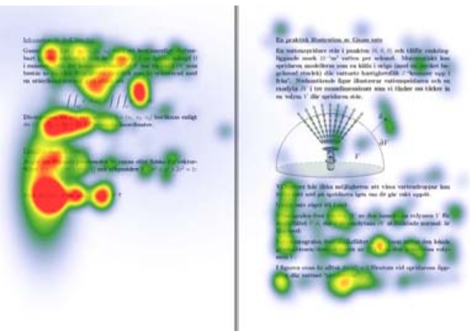


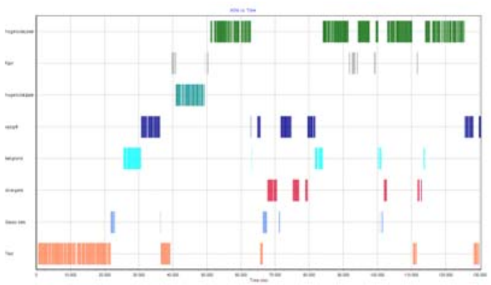


EYE-TRACKING DATA (x(t), y(t))

Quantitative (statistical) analyze

SOUND DATA





Qualitative analyze

Example (P6) of a recorded movie from the study:

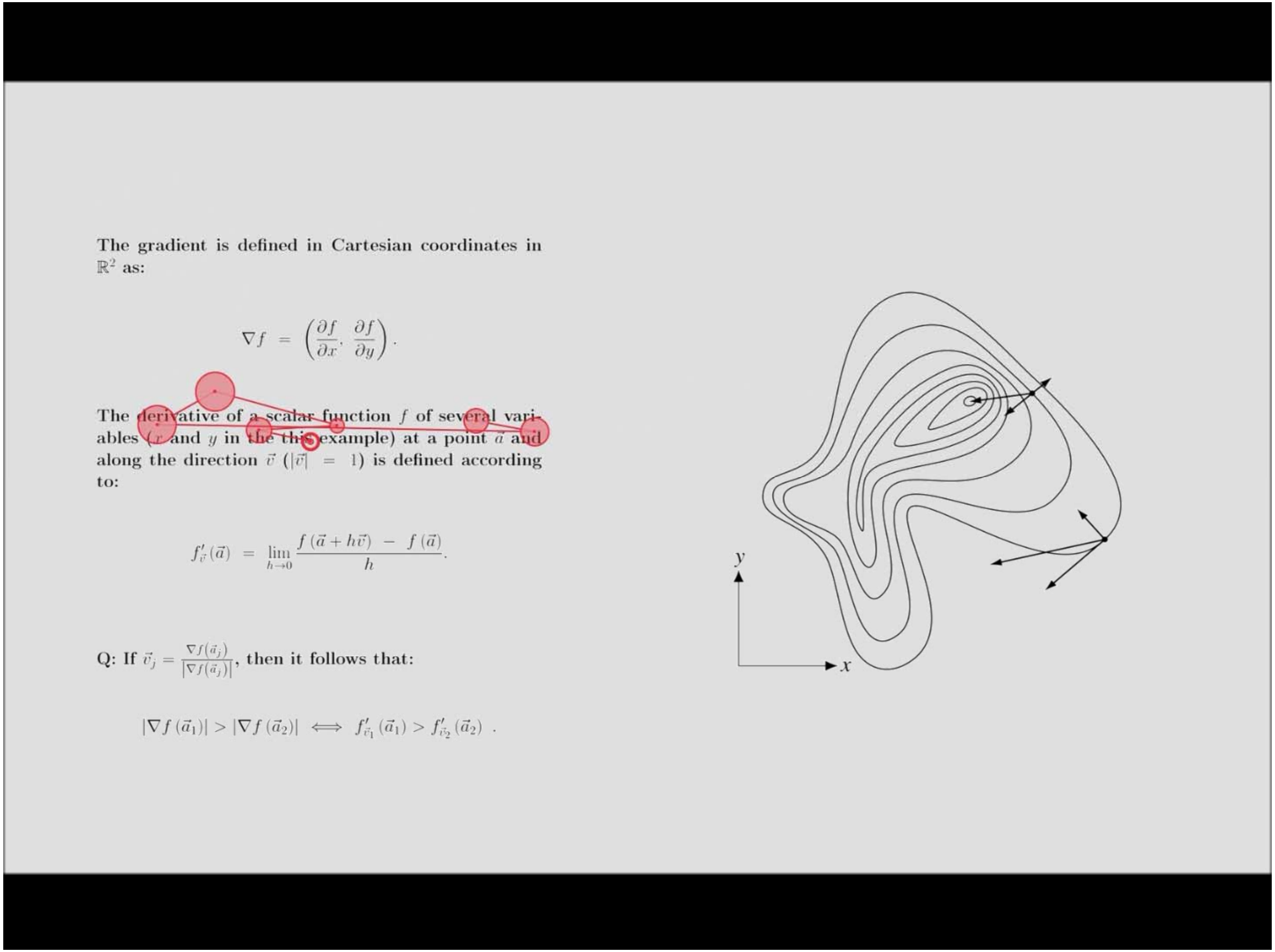
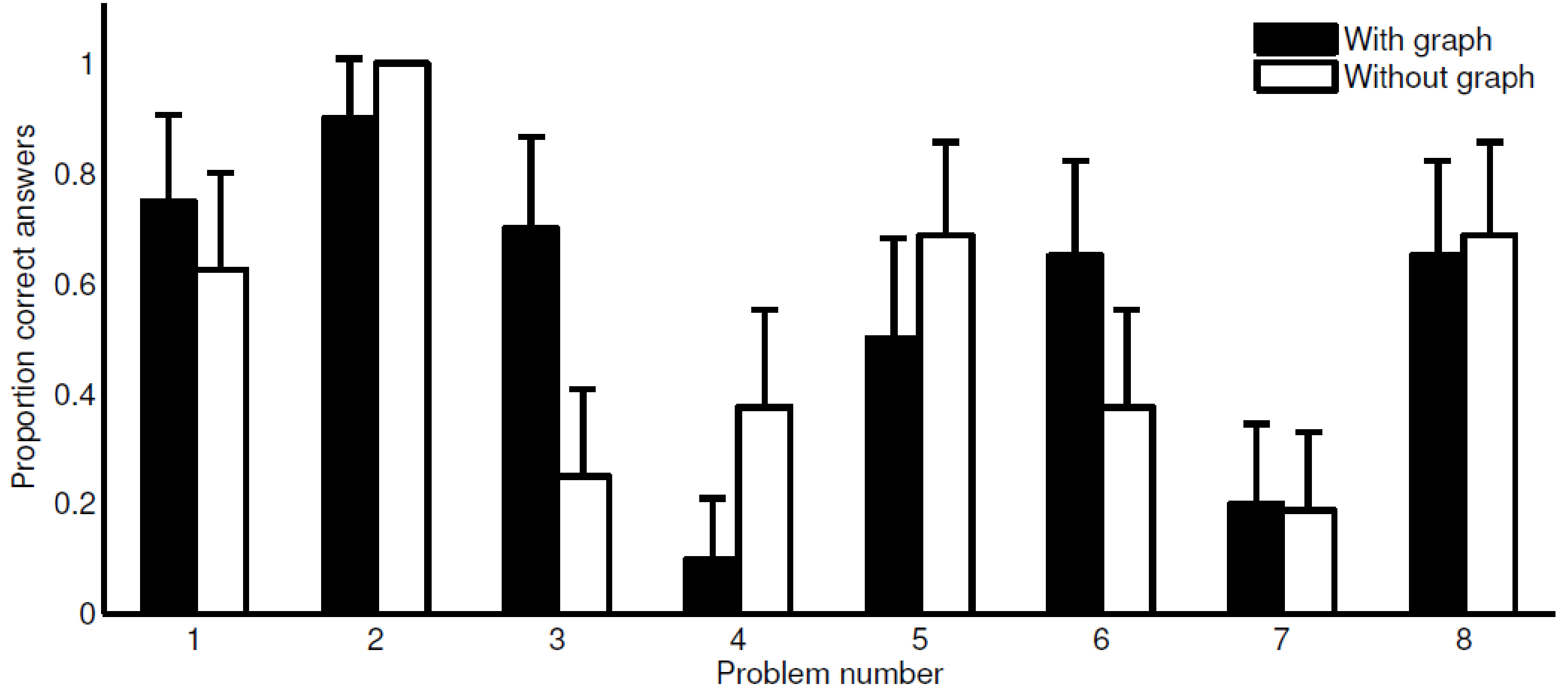


Table 1: Result of the multilevel logistic regression. Here 'condition 2' refers to the presentation without illustrations. The sign of the Estimate tells us that the condition with illustrations led to a higher number of correct answers, but the effect was not significant.

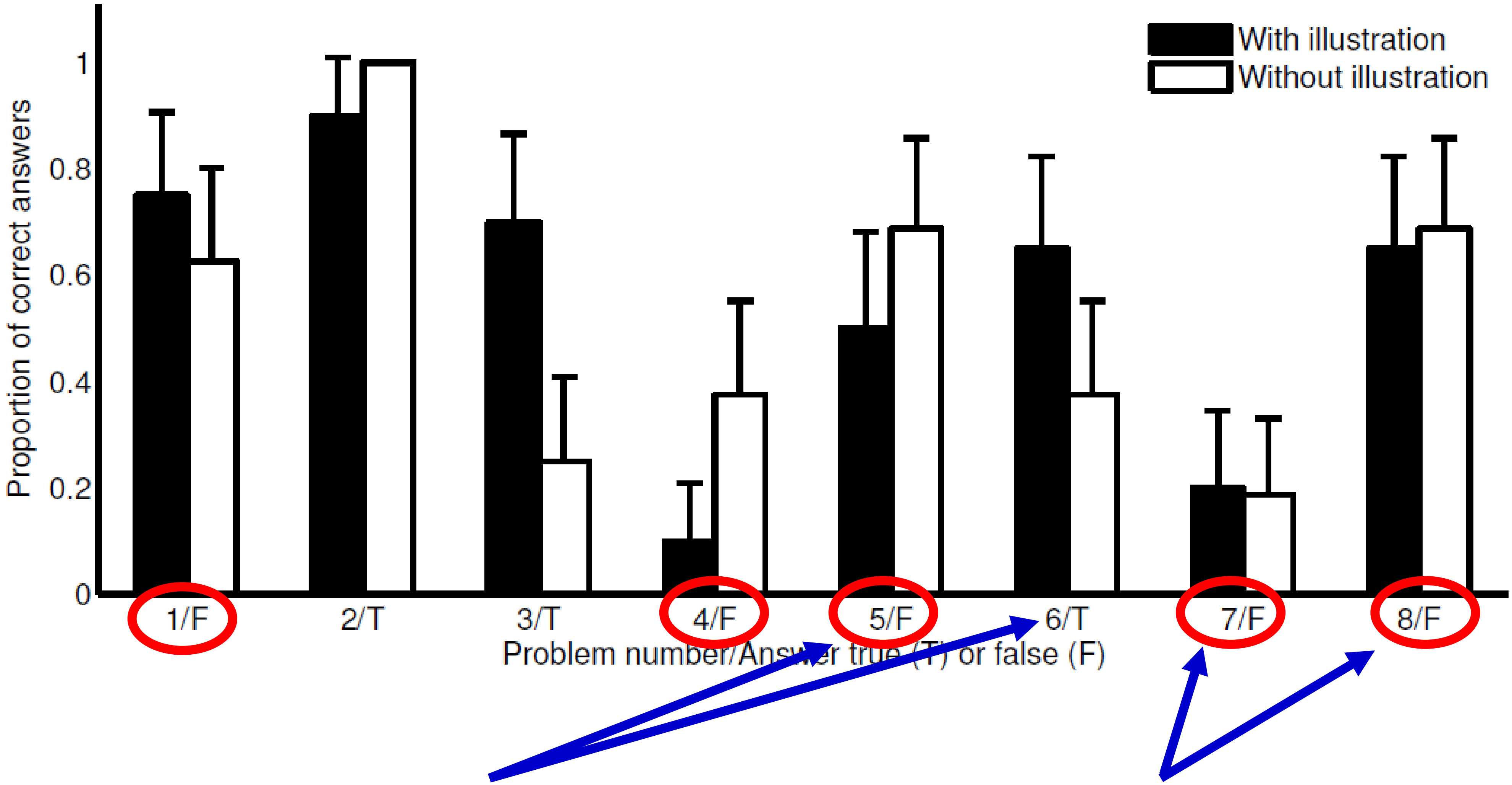
	Estimate	Std. Error	z value	Pr(>|z|)
(Intercept)	0.3019	0.4314	0.700	0.484
condition2	-0.1641	0.2622	-0.626	0.531

Overall an illustration helps but it is not significant.

There is strong variation between different problems



We note that the results are better when the correct answer is *true* (T), and in particular that an illustration gives worse results for questions where the answer is *false* (F).



Same figure (almost) **Standard textbook figures**

These claims are strongly supported by a statistical analysis:

	Estimate	Std. Error	z value	Pr(>|z|)
(Intercept)	-0.3132	0.5121	-0.612	0.540
withoutIllustration	0.3537	0.3285	1.077	0.281
answerTrue	1.7331	0.8578	2.020	0.043*
withoutIllustration:answerTrue	-1.4762	0.5625	-2.624	0.008**

What conclusions should we draw from this table?

*** Students at technical universities are bad in revealing false statements? (No evidence for lower ‘confidence’ in wrong answers!)**

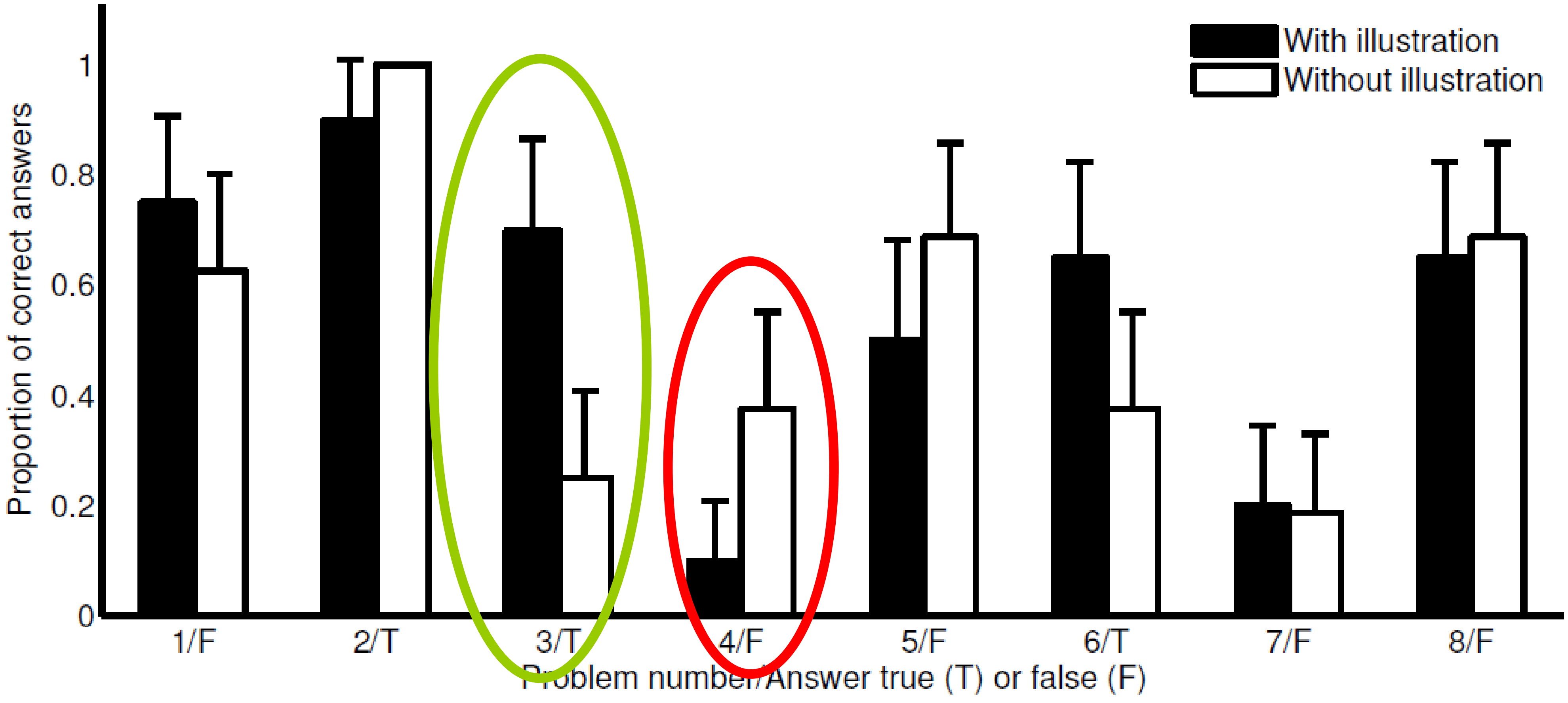
*** The design of the study with true/false questions does not very well correspond to the students normal situation?**

*** Import today to be able to deliver a (preliminary) answer to if a given information is true or false.**

Now follows some examples of analyses of individual problems.

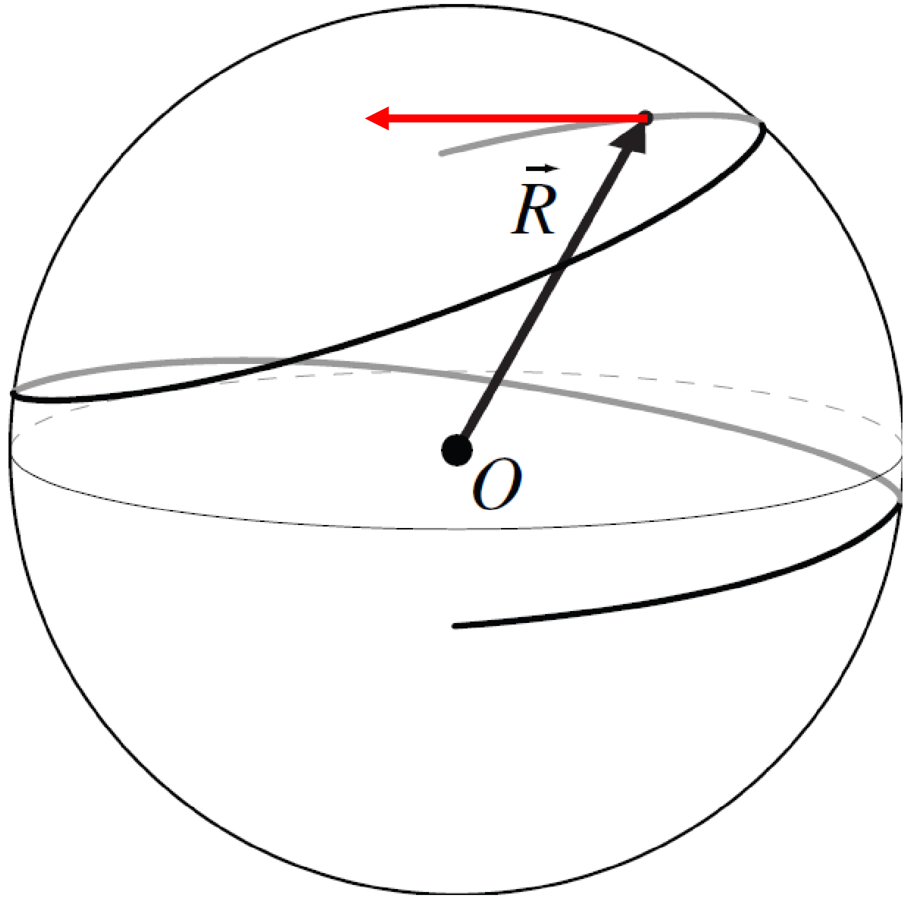
We have used eye movements and speech data to interpret strategies used by the students.

We choose to present only the two most extreme problems here:



Example of a figure that did help a lot in one out of two ways to solve the problem:

The red arrow (not shown for the students) resulting from the black arrow moving along the trajectory on the sphere indicate the solution of the problem.

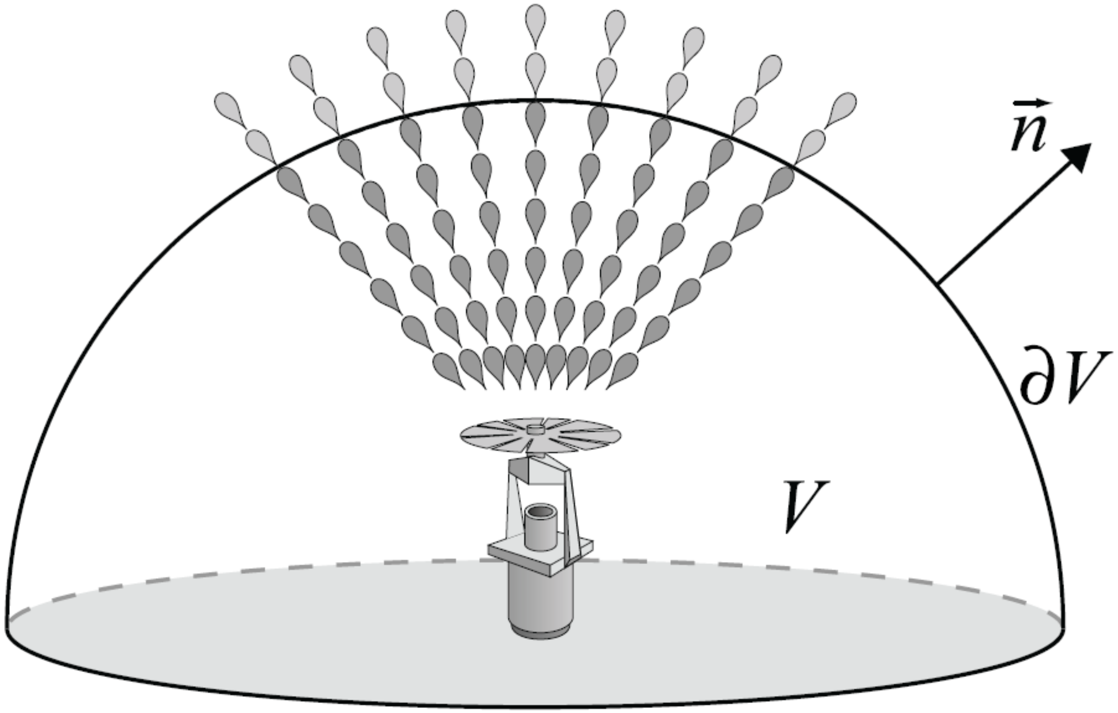


In the group with/without illustrations 0/1 did use an 'equation based' solution.

Only 1 person in the group without illustrations verbally described a sphere and solved the problem this way.

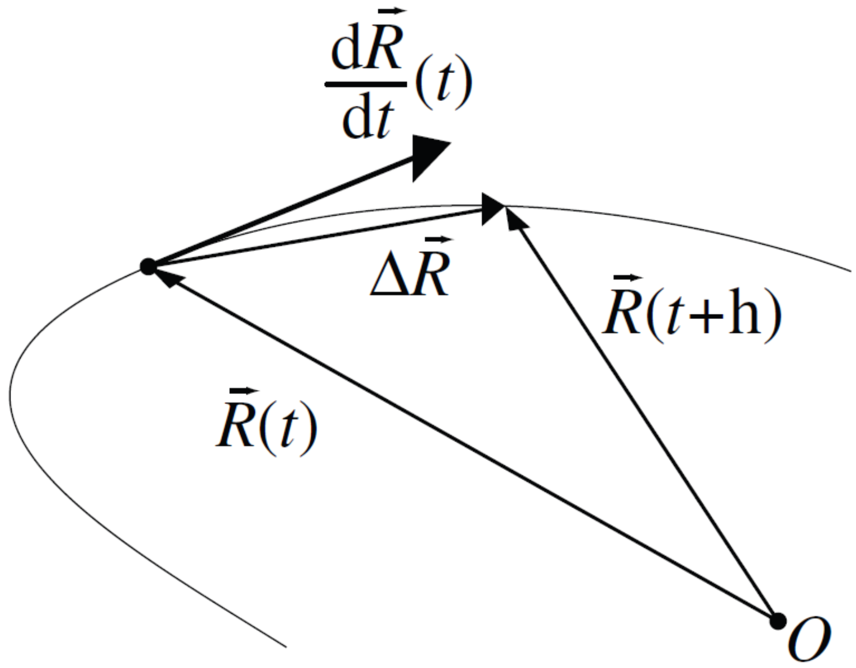
Hence, the strategy was here largely affected by an illustration.

Example of an illustration that did not help.
Instead it seemed to distract the students (less time fixating critical parts of the text).

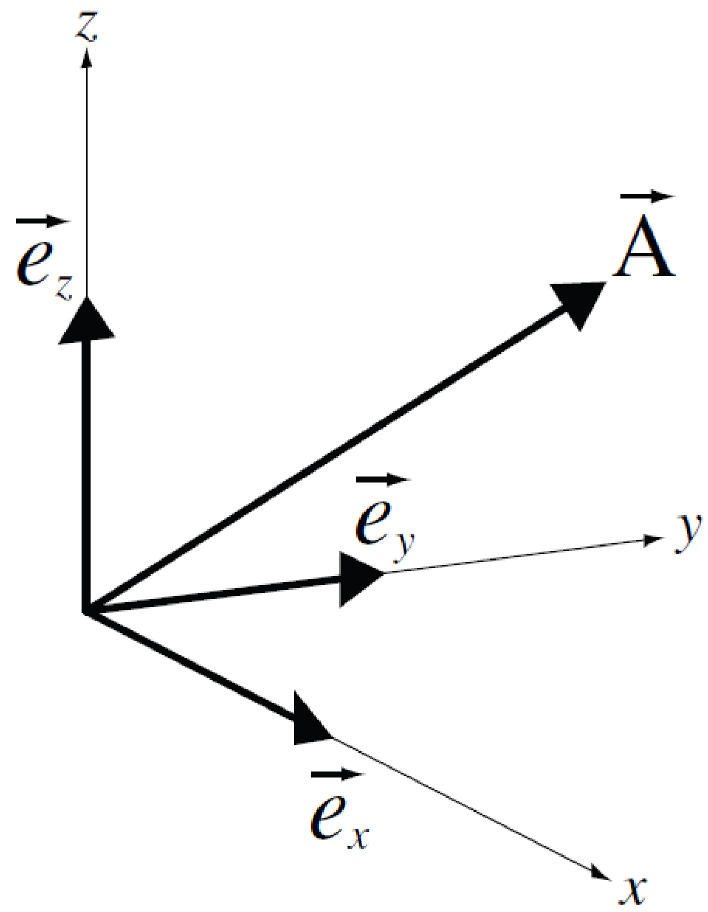


The illustration shows a possible application of Gauss formula.
Understanding the relation between the Gauss formula and the illustration misled many participants to give a *true* answer.

Other examples of typical standard textbook illustrations that did not have any positive effect on performance here:

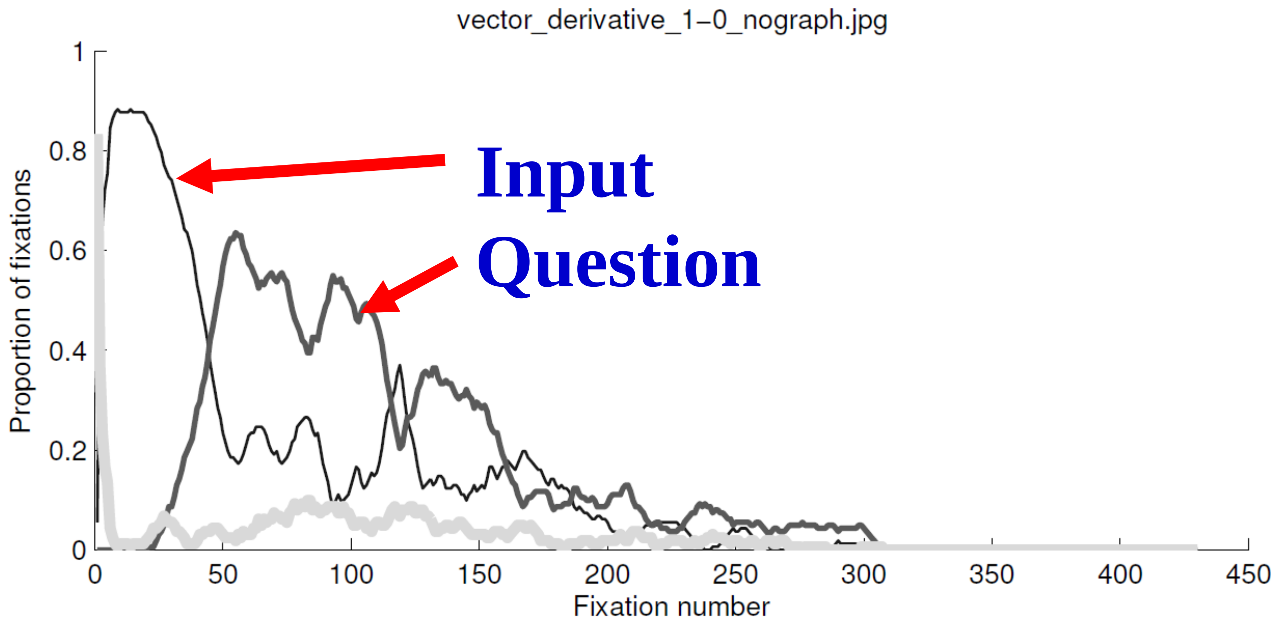


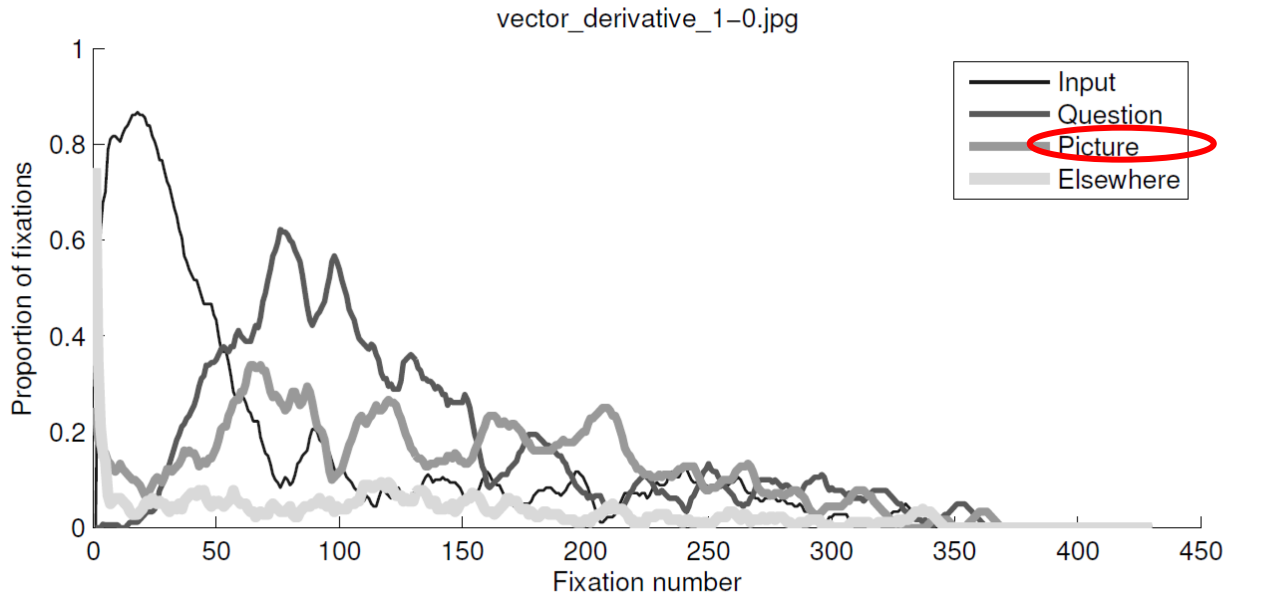
(P7) Illustration related to the derivative of a vector.



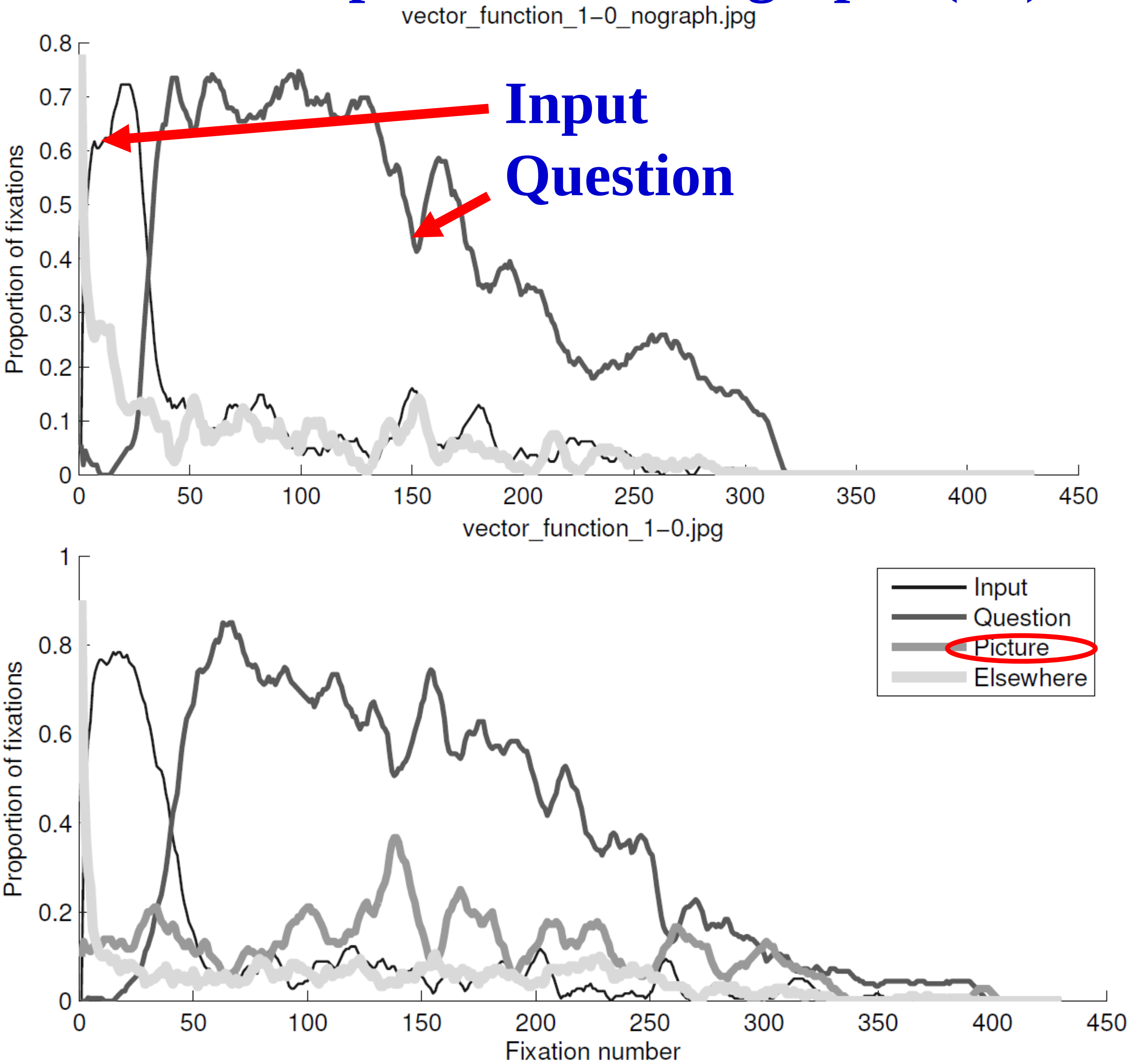
(P8) Illustration related to the length of a vector in terms of its components.

Proportion of time graphs (P7).





Proportion of time graphs (P8).



Conclusions from the study:

Hypothesis/Questions: **”A figure always helps!” ?**

Choose illustrations with care!

*** How do the illustrations affect comprehension?**

Supports successful graphical approach (e.g. P3).

Can give an overload of impressions (e.g. P4).

*** Effects on the allocation of visual attention?**

Distribution of input and question is not effected.

Minor decrease in favor of the illustration.

Future work:

****We are working on the analyse of pupil size data as an indicator of cognitive load, and on the dynamics when reading symbols and formulas.***

****How to use future eye-tracking equipment (digital classroom and cheap laptop cameras) as an integrated part of everyday teaching and examination?***

THANKS FOR YOUR ATTENTION!

Please: Any comments and ideas are welcome!

References:

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