FINITE-SIZE EFFECTS IN ATOM-ATOM CORRELATIONS IN MOLECULAR DISSOCIATION

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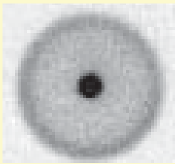
Motivation

Dissociation of a BEC of molecular dimers can produce pair-correlated atomic ensembles, with either fermionic or bosonic atom statistics, experiment e.g. at JILA [1].

Questions:

- How does the spatial inhomogeneity of the molecular BEC affects the strength of atom-atom correlations?
- What is the correlation width?

Due to momentum conservation the two atoms are of equal but opposite momenta, $-\mathbf{k}$ and $\mathbf{k}$.



We compare our results for non-uniform initial molecular BEC with those obtained in idealised uniform systems.

Effective Quantum Field Theory Model

The coupled molecular ($\hat{\Psi}_0$) and atomic ($\hat{\Psi}_\sigma$) fields can be described by

$$\hat{H} = \hat{H}_0 - i\hbar\chi \int d\mathbf{x} \left(\hat{\Psi}_0^\dagger \hat{\Psi}_\uparrow \hat{\Psi}_\downarrow - \hat{\Psi}_\downarrow^\dagger \hat{\Psi}_\uparrow^\dagger \hat{\Psi}_0 \right),$$

where $\sigma = \uparrow, \downarrow$ denotes different spin states (in the bosonic case one can also have dissociation into only one atomic state).

Within the undepleted molecule approximation, $\langle \hat{\Psi}_0(\mathbf{x}, t) \rangle \rightarrow \sqrt{n_0(\mathbf{x})}$, the Heisenberg equations for creation-/annihilation operators of plane wave modes for the ($-/+$ fermionic-/bosonic-) atoms are

$$\frac{\partial \hat{a}_{\mathbf{k},\uparrow}}{\partial t} = -i\Delta_{\mathbf{k}} \hat{a}_{\mathbf{k},\uparrow} \mp \frac{1}{L^{D/3}} \sum_{\mathbf{k}'} G_{\mathbf{k}'+\mathbf{k}} \hat{a}^\dagger_{\mathbf{k}',\downarrow},$$

$$\frac{\partial \hat{a}^\dagger_{\mathbf{k},\downarrow}}{\partial t} = i\Delta_{\mathbf{k}} \hat{a}^\dagger_{\mathbf{k},\downarrow} + \frac{1}{L^{D/3}} \sum_{\mathbf{k}'} G_{\mathbf{k}'-\mathbf{k}} \hat{a}_{-\mathbf{k}',\uparrow},$$

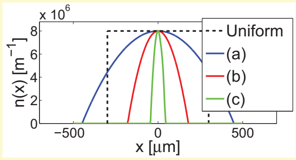
where $\Delta_{\mathbf{k}} = \Delta + \hbar \mathbf{k}^2 / (2m_{at})$, L is the size and D the dimension of the system. Here $G_{\mathbf{k}}$ is the fourier transform of $\chi\sqrt{n_0(\mathbf{x})}$.

Uniform Molecular Field

The case where $G_{\mathbf{k}} = G_0,\ else\ 0$ can be solved analytically for bosonic/fermionic atoms, within the undepleted molecule approximation [2]. See the dashed black lines in the following four figures.

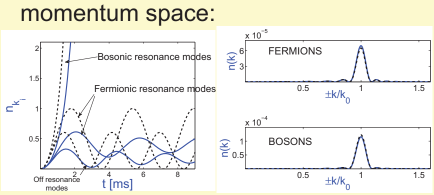
Non-uniform Molecular Field

For bosons, first principal calculations (+P) have been carried out [3]. For fermions we have done calculations (systems of ODE:s) both for the undepleted molecular approximation and for a time-dependant single mode (uniform case).

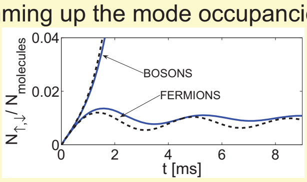


Mode occupancies and atom populations

We simulate occupancy dynamic in momentum space:



The number of atoms are obtained by summing up the mode occupancies:



Correlation Functions

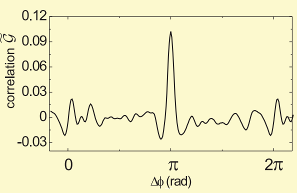
To study correlations in momentum space the following definition is made:

$$g_{\uparrow,\downarrow}(\mathbf{k}, \mathbf{k}', t) = \frac{\langle \Delta\hat{n}_{\mathbf{k},\uparrow} \Delta\hat{n}_{\mathbf{k}',\downarrow} \rangle}{\sqrt{\langle \hat{n}_{\mathbf{k},\uparrow} \rangle \langle \hat{n}_{\mathbf{k}',\downarrow} \rangle}}.$$

Again here the uniform case can be done analytically within the undepleted molecular approximation $g_{\uparrow,\downarrow}(\mathbf{k}, \mathbf{k}' = -\mathbf{k}, t) = 1 \mp n_{\mathbf{k}}(t)$ ($-/+$ fermions/bosons), it has zero width i.e. $g_{\uparrow,\downarrow}(\mathbf{k}, \mathbf{k} \neq -\mathbf{k}', t) = 0$.

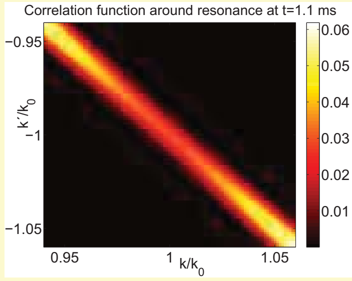
For a non-uniform molecular field, several momentum states start to couple, resulting in a finite width for the correlation function.

At JILA one have studied similar correlations from a cigar-shaped $^{40}K_2$ BEC [1]. Experiments with $^6Li_2$ are in progress at Swinburne :)



1D Simulations

Narrow molecular condensate, **(c)**:



Cut at the resonance $k = -k' = k_0$:

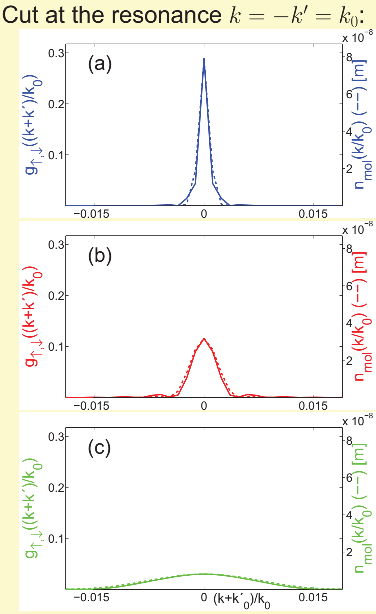
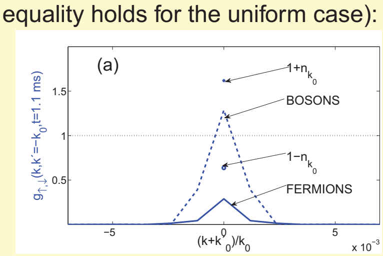


Illustration of the inequality: $g_{\uparrow,\downarrow}(\mathbf{k}, \mathbf{k}' = -\mathbf{k}, t) \leq 1 \mp n_{\mathbf{k}}(t)$ (the equality holds for the uniform case):



Summary

The width of the correlation function becomes broader for a narrower molecular field, more specifically the correlation function and the molecular density in momentum space are proportional and can be accurately described in terms of a Bessel function: $g \propto J_1^2(R_{TF}k)/k^2$, of width $\simeq 3.2/R_{TF}$.

References

[1] M. Greiner *et al.*, Phys. Rev. Lett. **94**, 110401 (2005).

[2] K.V. Kheruntsyan, Phys. Rev. Lett. **96**, 110401 (2006).

[3] C.M. Savage *et al.*, Phys. Rev. A **74**, 033620 (2006).