Role of Spatial Inhomogeneity in Dissociation of Trapped Molecular Bose-Einstein Condensates (MBEC)

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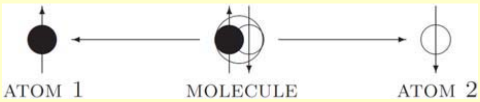
Motivation

Pairs of correlated fermionic atoms can be produced by dissociation of molecular Bose-Einstein condensates of dimers [1]. This process represents a matter-wave analogue of two-photon parametric down conversion, which has been pivotal in the advancement of quantum optics with photons.

- What types of fermion-fermion correlations do we get from molecular dissociation?
- How does the spatial inhomogeneity of the MBEC affects the strength and shape of the atom-atom correlations?
- Future atom optics demonstrations of Einstein-Podolsky-Rosen entanglement and related tests of Bell's inequalities?

Model system

Each individual dissociation process fulfills conservation of momenta in the center of mass frame of the dimer molecule.



- The momentum conservation is the physical source of the so called back-to-back (BB) atom-atom correlation.
- We also expect collinear (CL) correlations due to many-body quantum statistical effects.

Model Hamiltonian

$$\widehat{H} = \int d^D\mathbf{x} \left\{ \sum_{j=1,2} \left(\frac{\hbar^2}{2m} |\nabla \widehat{\Psi}_j|^2 + \hbar\Delta \widehat{\Psi}_j^\dagger \widehat{\Psi}_j \right) - i\hbar\chi\sqrt{\rho(\mathbf{x})} \left(\widehat{\Psi}_1 \widehat{\Psi}_2 - \widehat{\Psi}_2^\dagger \widehat{\Psi}_1^\dagger \right) \right\}$$

- $\rho(\mathbf{x})$ denotes the spatial density of the MBEC.
- The atomic field operators $\hat{\psi}_j(\mathbf{x})$ of spin-state $j=1,2$ obey either fermionic or bosonic commutation relations.
- We have mainly studied atom-atom correlations for molecules made of fermionic atoms [2,3].

Implementation

From the quadratic model Hamiltonian, we obtain a set of linear Heisenberg operator equations for the Fourier components of $\hat{\psi}_j$

$$\frac{d\widehat{a}_1(\mathbf{k},t)}{dt} = -i\Delta_k \widehat{a}_1(\mathbf{k},t) \pm \int \frac{d^D\mathbf{q}}{(2\pi)^{D/2}} \widetilde{g}(\mathbf{q}+\mathbf{k}) \widehat{a}_2^\dagger(\mathbf{q},t)$$

Here - (+) is for fermions (bosons), with a similar equation for $\hat{a}_2$. The density profile of the MBEC $\rho(\mathbf{x})$ is encoded via its Fourier transform $\widetilde{g}(\mathbf{k})$.

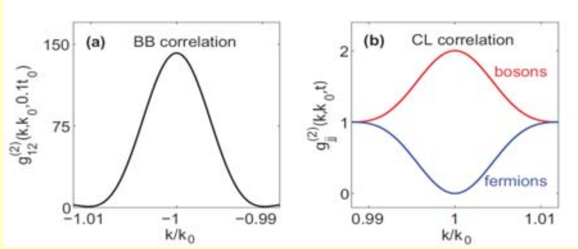
In the short time limit, the atomic annihilation (and creation) operators can be Taylor expanded in time

$$\widehat{a}_j(\mathbf{k},t) = \widehat{a}_j(\mathbf{k},0) + t \frac{\partial \widehat{a}_j(\mathbf{k},t)}{\partial t}\bigg|_{t=0} + \frac{t^2}{2} \frac{\partial^2 \widehat{a}_j(\mathbf{k},t)}{\partial t^2}\bigg|_{t=0} + \ldots$$

This leads to analytic short-time asymptotes for correlation functions. For long times, solutions of the Heisenberg equations can be calculated via numerical matrix exponentiation. The results here are obtained within the undepleted molecular field approximation. While only accurate for less than 10% conversion, this is often a useful approach for fermions, where Pauli blocking prevents large depletion of the MBEC.

Atom-atom correlations

A Thomas-Fermi density profile of size $R_{TF,i}$ ($i=x,y,z$) for the source MBEC, has a momentum distribution of width $\sim 1/R_{TF,i}$. The width of the atom-atom momentum correlations is determined by the momentum width of the source $\sim 1/R_{TF,i}$.



Analytic results; (a) back-to-back (BB) (with $g^{(2)}_{BB}(\mathbf{k},-\mathbf{k})>2$)

$$g_{12}^{(2)}(k,k',t) \simeq 1 + \frac{9\pi^2}{16t^2\chi^2\rho_0} \frac{\left[J_1\left((k+k')R_{TF}\right)\right]^2}{\left[(k+k')R_{TF}\right]^2}$$

and (b) collinear (CL) correlation [3]

$$g_{jj}^{(2)}(k,k',t) \simeq 1 \pm \frac{9\pi}{2} \frac{\left[J_{3/2}\left((k-k')R_{TF}\right)\right]^2}{\left[(k-k')R_{TF}\right]^3}$$

Collinear correlation (CL) is a quantum statistical effect with:

- Pauli-blocking for fermionic atoms (-), and $g^{(2)}_{CL}(\mathbf{k},\mathbf{k})=0$.
- Bose-stimulation in the bosonic case (+), and $g^{(2)}_{CL}(\mathbf{k},\mathbf{k})=2$.

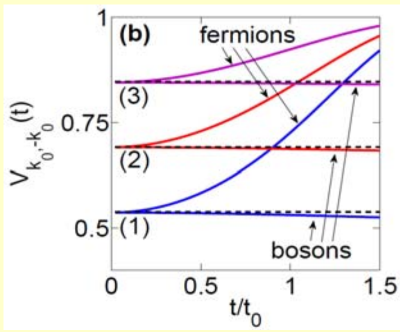
Relative number squeezing

Other measures of correlations:
normalised variance of number-difference fluctuations:

$$V_{k_0,-k_0}(t) = \langle [\Delta(\widehat{n}_{1,k_0} - \widehat{n}_{2,-k_0})]^2 \rangle / \Delta_{\mathrm{SN}}$$

$$V_{k_0,-k_0}(t) \simeq 1 - 3\pi\Delta k R_{TF}/32$$

- $V<1$ implies squeezing of fluctuations below the standard quantum limit.
- Dashed lines – analytic results, solid – numerics
- Sizes of 1D MBECs:
  (1) R_{TF}=250 μm
  (2) R_{TF}=167 μm
  (3) R_{TF}=83 μm
- A broader MBEC gives stronger number squeezing



Summary

- ***We have found analytic expressions for the strength and shape of atom-atom momentum correlations.***
- ***While strictly valid in the short time limit ($t/t_0 \ll 1$), they agree qualitatively with numerics at larger times ($t/t_0 \sim 1$).***
- ***Our analytic results give useful physical insights into the dynamics of molecular dissociation.***

Future work

- Include molecular depletion and atom-atom *s*-wave interactions, using stochastic phase-space methods for ab-initio dynamics.

[1] M. Greiner *et al.*, Phys. Rev. Lett. **94**, 110401 (2005).
[2] M. Ögren and K. V. Kheruntsyan, Phys. Rev. A **78**, 011602(R) (2008).
[3] M. Ögren and K. V. Kheruntsyan, ArXiv:0905.0343.