Fermionization of an interacting 1D Bose gas

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Introduction

- The Gross-Pitaevskii equation (GPE) is expected to give a good description of 1-D BECs near $T = 0$ in the regime of weak interactions.
- However for strong interactions (Tonk-Girardeau-regime) the Bose gas has 'fermionic' properties and can be described as a non-interacting Fermi gas via the Fermi-Bose-Mapping-Theorem [1].

Homogeneous Bose Gases

We use a numerical solution to Lieb and Liniger's exact state-equation for 1-D Bose gas interacting via a adjustiable $\delta_c(x_i - x_j)$-potential [2]: The zero temperature ground energy with N atoms and the uniform particle density $n = N/L$ is[a]:

$$E_0 = Nn^2 e(\gamma)\ ,\ \gamma = c/n$$

Where the function $e(\gamma)$ is the solution to a system of equations, that can't be solved analytically.
We can directly presume that:

- $e(0) = 0$ since non-interacting ($c = 0$) free bosons have zero ground energy.
- $e(\infty) = \pi^2/3$ since this is the result for fermions ($c \to \infty$).

[a] We use units where $\hbar = 2m = 1$, c is then related to the 3D scattering length according to: $c = 2a_{3D}/l_\perp^2$

Local Density Approximation

If the variation in particle-density is small compared to the average interparticle distance Lieb and Linigers result can be applied locally: $E_0/N = n(x)^2 e(\gamma(x))$

Hydrodynamic Equations

We have been simulating non-linear Schrödinger like hydrodynamic equations:

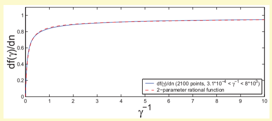
$$i\frac{\partial \Psi}{\partial t} = \left[-\frac{\partial^2}{\partial x^2} + 2cf(n) + V_{ext}\right]\Psi$$

where $n = |\Psi(x,t)|^2$. Now we apply LDA:

$$f(n) = \frac{1}{2c} \cdot \frac{d[e(\gamma)\cdot n^3]}{dn}$$

Since $e(\gamma)$ can't be expressed analytically, we fit a 2-parametric rational function to the derivaty of $f(n)$:

$$f'(\gamma) \approx \frac{1 + c_1 \cdot \pi^2 \gamma}{1 + c_2 \cdot \gamma + c_1 \gamma^2}$$



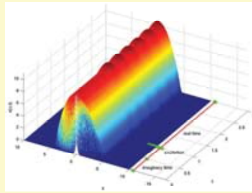
After an integration we then obtain the 'interaction function':

$$f(n) \approx n - \ln\left[\left(1 + \frac{n}{A}\right)^\alpha \left(1 + \frac{n}{B}\right)^\beta\right]$$

where A, B, α, β are numerical constants which depend on c_1, c_2 and c.

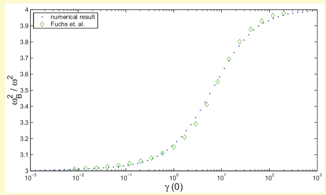
Numerical Results

We have used consecutive integration sections in XMDS [3] to first obtain a ground-state, with an imaginary-time method, and when performed an excitation in real-time by strengthen the potential:



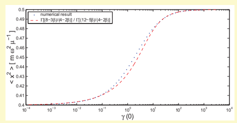
Dynamic properties

We have calculated the square of the ratio between the oscillation frequency of the (first) breathing mode, ω_B, and the strength of a harmonic potential, ω, which undergoes a transition[a] from 3 to 4:



Static properties

The 'normalized' second moment:
$\langle x^2 \rangle \cdot [m\omega^2\mu^{-1}]$
Undergoes a transition[a] from 0.4 to 0.5:



[a] The limits corresponds to the results from the Gross-Pitaevskii- respective Kolomeisky-Straley-equations.

Conclusions

- We have developed methods to simulate static and long wavelength dynamical properties of 1-D Bose gases, relevant to current experiments in optical lattices and atom-chips.
- Results for the breathing mode ratios are consistent with [4] and [5], which have used different methods.
- Gives us confidence that calculations of other dynamical properties can be done with this method.
- For further details see my diploma work report: *www.magnus.ogren.se/report.pdf*

Outlook

- Can we generalise the method to finite temperatures?
- Future applications to trapped 1-D gases on chips?

References

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