Superfluidity versus disorder in a BEC

M. Ögren and G. M. Kavoulakis

Mathematical Physics LTH, Lund University, P.O. Box 118, SE-221 00 Lund, Sweden

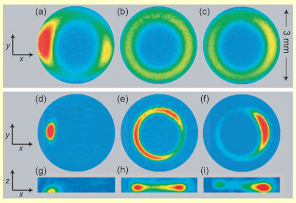
Lund Institute of Technology
Lund University

Introduction

Motivated by recent experiments which have managed to trap Bose-Einstein condensed atoms in toroidal traps [1], we examine the effect of disorder on the metastability of supercurrents in these systems [2]. We restrict ourselves to the limit of relatively weak interactions and relatively weak disorder, where the mean-field approximation is expected to provide a good description of the order parameter. Also, we model the disorder as a piecewise constant potential with a random amplitude. The length scale of variation of the potential is chosen to be a fraction (1/10) of the radius of the torus, as shown in Fig. 3. The reader who is interested in the stability of supercurrents as one goes from the (mean-field) limit of weak interactions, to the Tonks-Girardeau limit of hard-core bosons (in the absence of disorder), should look at Ref. [3].

Toroidal traps: now realized experimentally

Figure 1: Recent experiments [1] have designed toroidal traps, and have even loaded Bose-Einstein condensed atoms in them. Such traps with a tight transverse confinement make it possible to realize quasi one-dimensional systems with periodic boundary conditions.



A toy-model for persistent currents

The Gross-Pitaevskii equation for the quasi-one-dimensional system that we consider in this study is (in atomic units, $\hbar = 2M = R = 1$),

$$i\partial_t \Psi = \left[-\partial_\theta^2 + 2\pi\gamma|\Psi|^2 + V\right]\Psi. \qquad (1)$$

Here γ is the strength of the coupling constant between the atoms, which is assumed to be positive (for repulsive interactions).
To get some insight into the metastability of superflow, we consider the trial order parameter [4]

$$\Psi = c_0\phi_0 + c_1\phi_1 = \sqrt{1-\ell}\,\phi_0 + e^{i\lambda}\sqrt{\ell}\,\phi_1, \qquad (2)$$

where $\phi_m = e^{im\theta}/\sqrt{2\pi}$ are the usual plane wave states and λ is some phase. The above state has by construction ℓ units of angular momentum per atom, with $0 \leq \ell \leq 1$.

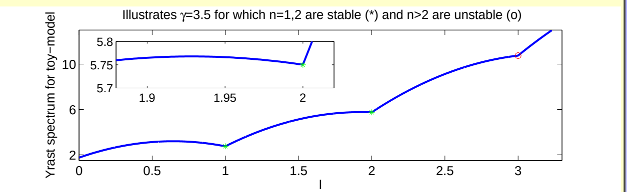
Energy barriers in the absence of any potential along the torus

In the absence of any external potential ($V = 0$), the energy of the system within the Gross-Pitaevskii, mean-field approximation is

$$E = \int d\theta \left\{|\partial_\theta\Psi|^2 + \pi\gamma|\Psi|^4\right\} = \gamma/2 + \ell\,(1+\gamma) - \gamma\ell^2. \qquad (3)$$

The dispersion relation of Eq. (3) develops a local maximum at $\ell = (1+\gamma)/(2\gamma) \leq 1$, provided that the coupling constant exceeds the critical value $\gamma_c = 1$. Beyond this γ, the state with one unit of circulation becomes metastable, as there is an energy barrier that the system has to overcome to relax to the non-circulating state. These arguments can be generalized for any circulation that is an integer multiple of h/M, as shown in Fig. 2.

Figure 2:



Introducing a (random) potential

An external potential $V(\theta)$ contributes an additional term to the energy

$$\langle V\rangle = \int V(\theta)|\Psi|^2\,d\theta = \frac{1}{2\pi}\int V(\theta)\left\{1+2\sqrt{\ell\,(1-\ell)}\cos(\theta+\lambda)\right\}d\theta, \qquad (4)$$

where the freedom to choose the phase $\lambda = 0$ or π guarantees that the second term is always negative and thus reduces the energy. Collecting all the terms we get that the total energy is,

$$E_{tot} - \gamma/2 - \int V(\theta)\,d\theta = \ell\,(1+\gamma) - \gamma\ell^2 - \sqrt{\ell\,(1-\ell)}|V_c|/\pi, \qquad (5)$$

where $V_c = \int V(\theta)\cos(\theta)\,d\theta$. The potential destabilizes the current. In order for this to be stable, the coupling needs to be higher than $\gamma_c = 1$. More generally, if one considers a potential $\kappa V(\theta)$ with κ being a dimensionless constant which is the "strength" of the disorder, γ_c is an increasing function of κ. The lower curve in Fig. 4 shows $\gamma_c(\kappa)$ calculated within the toy model described above.

Numerical results

To get some realistic results, we examine the stability of the state with one unit of circulation numerically, via the method of imaginary-time propagation of the Gross-Pitaevskii equation (1), using the software XMDS [5]. More specifically, we examine the critical value of the coupling constant that gives stability of the state $\phi_1 = e^{i\theta}/\sqrt{2\pi}$ for a given potential $\kappa V(\theta)$. The result of this calculation is shown as the solid curve in Fig. 4.

Figure 3 : The random potential that was used to calculate $\gamma_c(\kappa)$ in Fig. 4 (with $\kappa = 1$).

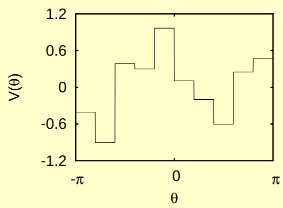
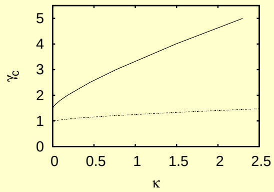


Figure 4: The solid line shows $\gamma_c(\kappa)$ calculated numerically, within the method of imaginary-time propagation. The dashed curve shows the same function calculated within the toy model described above.



Finally, a Bogoliubov transformation gives the whole excitation spectrum as well as the depletion of the condensate, however it does not give the dependence of γ_c on κ, for more details, see Ref. [2].

Remarks and conclusions

Any (meta)stable current that circulates in a toroidal trap becomes more fragile in the presence of an external potential. This is simply due to the fact that the variation in the atomic density that results from the external potential makes it energetically less expensive for the system to get rid of the circulation, as compared to the case of a homogeneous system. As a result, strong disorder requires a correspondingly strong coupling to stabilize the current.

References

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