Quantum correlations in molecular dissociation to fermion pairs

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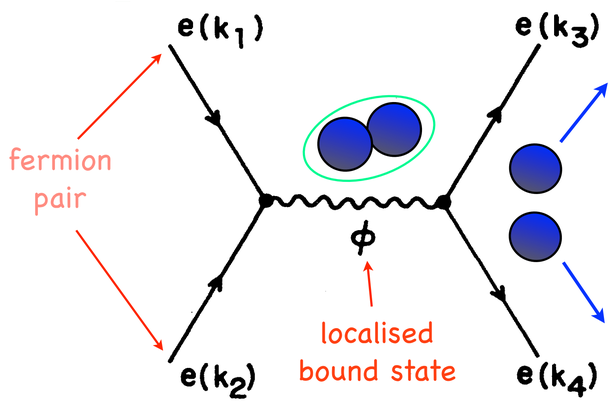
Abstract

We calculate the growth of correlations in a Fermi gas formed by dissociation from a molecular condensate, following the rapid quench through a Feshbach resonance. The exact quantum many-body dynamics are numerically simulated by means of a Gaussian phase-space representation. We quantify deviations of atom-atom pair correlations from Wick's factorization scheme, and show that atom-molecule and molecule-molecule correlations grow with time, in clear departures from pairing mean-field theories.

System

Ultracold molecules and atom pairs



Friedberg & Lee '93

Matter-wave analogue of spontaneous down-conversion in optics, the workhorse of entangled photon production in quantum optics, except that constituents are **fermions**.

Bose-Fermi Model

Fermions: $\hat{n}_{\mathbf{k},\sigma} = \hat{c}^\dagger_{\mathbf{k},\sigma}\hat{c}_{\mathbf{k},\sigma}$ (number)

$\hat{m}_{\mathbf{k}} = \hat{c}_{\mathbf{k},1}\hat{c}_{-\mathbf{k},2}$ (pairing)

$$\widehat{H} = \hbar\sum_{\mathbf{k},\sigma}\Delta_{\mathbf{k}}\hat{n}_{\mathbf{k},\sigma} - i\hbar\kappa\sum_{\mathbf{k}}\left(\hat{a}^\dagger\hat{m}_{\mathbf{k}} - \hat{m}^\dagger_{\mathbf{k}}\hat{a}\right)$$

Bose-Fermi coupling strength

kinetic and dissociation energy: $\hbar^2|\mathbf{k}|^2/(2m_a) + \hbar\Delta$

boson amplitude

Assumptions:

- uniform system
- ground state molecular condensate
- dilute atoms: ignore s-wave interactions

Method

Gaussian phase-space representation

Formally, we expand the many-body density operator as a mixture of nonHermitian Gaussian operators. The expansion is sampled as stochastic trajectories in a generalised Gaussian phase space:

number correlations (normal fluctuations)

$$P(\alpha, \alpha^+, n, m, m^+)$$

coherence amplitudes

squeezing correlations (anomalous fluctuations)

Final equations have the structure of mean-field equations, with extra stochastic terms that give the many-body quantum correlations

Calculations

Correlations functions

Calculate a suite of dynamical observables to benchmark the method and probe the many-body correlations

Molecule-Molecule Correlation:

$$g^{(2)}_{mm} = \frac{\langle \hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a}\rangle}{\langle \hat{a}^\dagger\hat{a}\rangle^2}$$

= 1 for coherent
= 2 for thermal

Atomic Correlation:

$$W = \sum_{\mathbf{k}}\langle \hat{m}^\dagger_{\mathbf{k}}\hat{m}_{\mathbf{k}}\rangle \Big/ \sum_{\mathbf{k}}\left(|\langle \hat{m}_{\mathbf{k}}\rangle|^2 + \langle \hat{n}_{\mathbf{k}}\rangle^2\right)$$

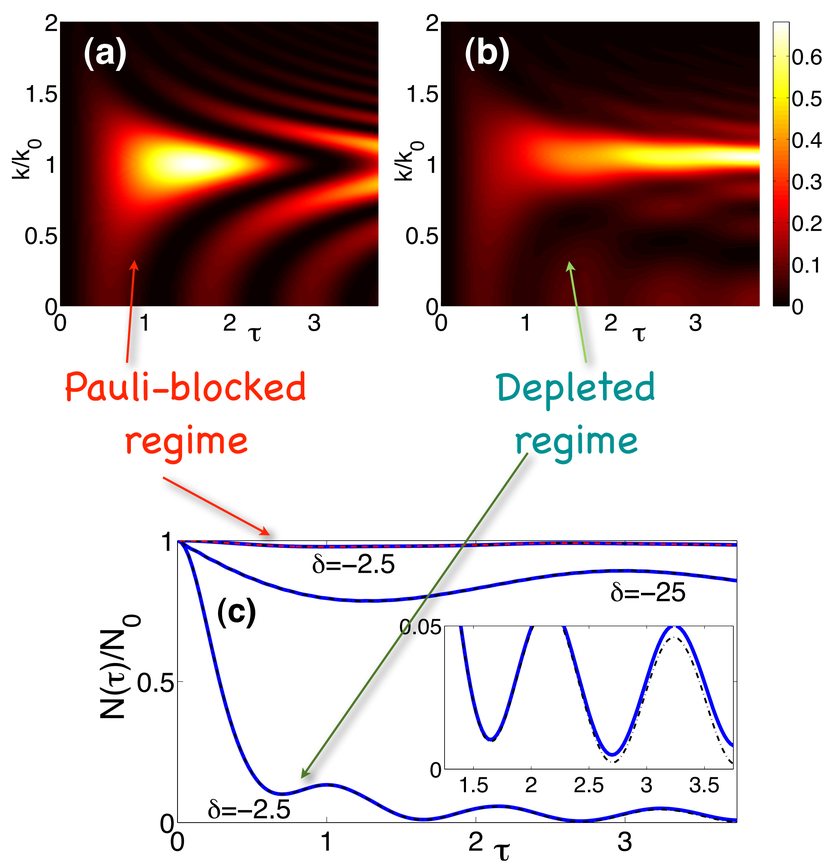
= 1 for Gaussian states, obeying Wick factorisation

The Gaussian phase-space methods allows us to determine the real-time evolution, without any assumptions about how correlations factorise.

1D Results

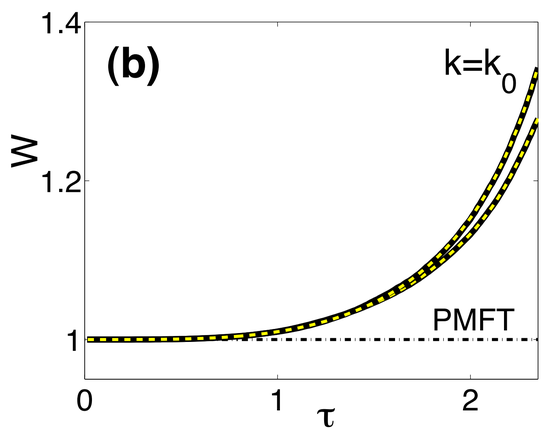
Quantum simulations

- prepare coherent-state molecular condensate
- sudden quench through Feshbach resonance
- watch molecules dissociate into atom pairs



Comparison to exact diagonalisation results

For small systems we can independently check the accuracy of the results (black lines) by comparisons with exact diagonalisation (dashed yellow):

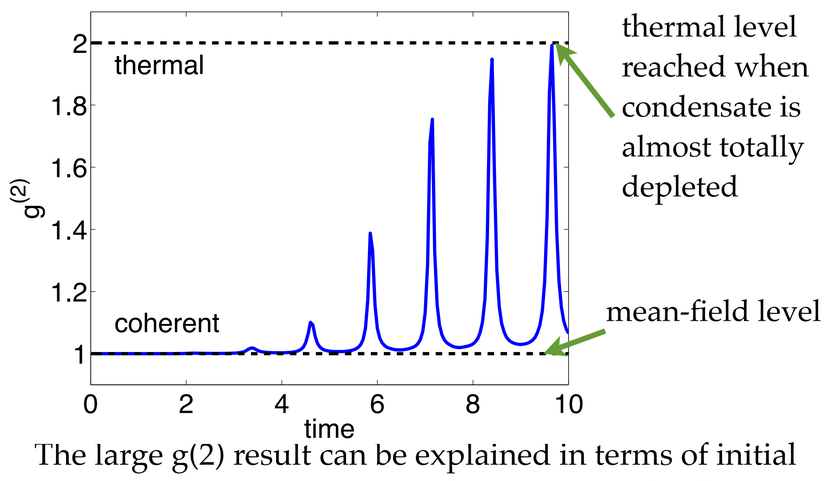


We see significant departures from mean-field results: Wick factorisation breaks down.

2D Results

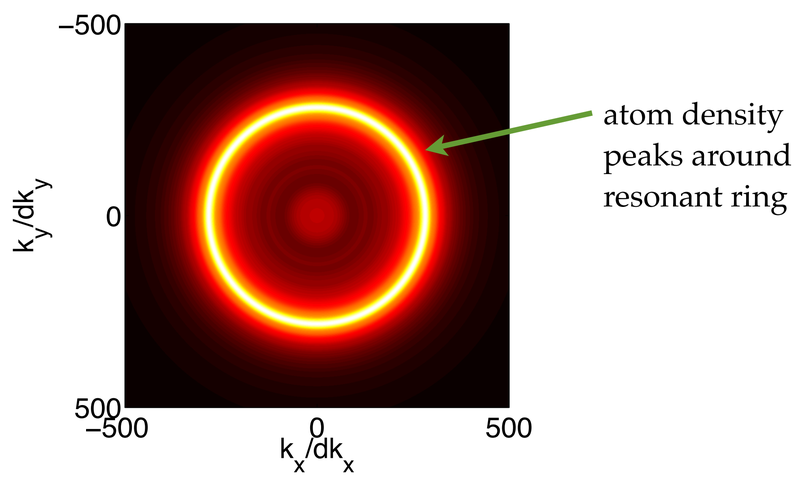
Dynamics in 2 dimensions

Molecular correlations

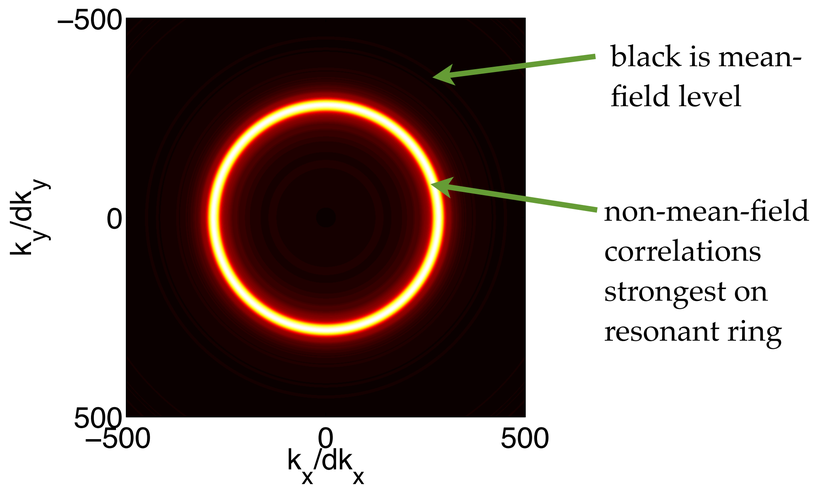


The large g(2) result can be explained in terms of initial uncertainties: superposition of oscillations of different frequency

Atomic momentum distribution at time = 10:



Atomic correlations W at time = 10:



Conclusions

Summary

The application of the Gaussian phase-space method gave simulations of useful duration of the quantum many-body dynamics, for large 1d and 2d systems..

We benchmarked the pairing mean-field theory, showing that it generally predicts populations accurately.

However, there were regimes where significant, higher-order correlations develop, unnaccounted for by mean-field theory.

We are currently testing the methods on the Hubbard model, with the aim of incorporating s-wave interactions into the dynamics

References

For more details:

M. Ögren, K. Kheruntsyan and J. F. Corney, *First-principles quantum dynamics for fermions: Application to molecular dissociation*, Europhysics Letters 92, 36003 (2010).

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