Stationary states and dynamics of superconducting thin films

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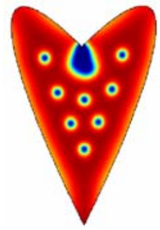


Motivation

The Ginzburg-Landau (GL) theory is a celebrated tool for microscopic modelling of superconductors (SC) [1]. We show examples from the use of the Finite Element Method (FEM) [2] for non-linear PDEs describing different aspects of GL theory.

Ginzburg-Landau Equation (GLE)

$$-\left(\tfrac{i}{\kappa}\nabla + \mathbf{A}\right)^2 \psi + \psi - |\psi|^2 \psi = 0,$$

$$\tfrac{1}{2i\kappa}\left(\psi^*\nabla\psi - \psi\nabla\psi^*\right) - |\psi|^2 \mathbf{A} - \nabla\times\nabla\times\mathbf{A} = 0.$$

For calculations of stationary states, for example of type-II SC with an external magnetic field. The initial condition is very crucial.

Time-Dependent Ginzburg-Landau Equation (TDGLE)

$$\partial_t\psi = -\left(\tfrac{i}{\kappa}\nabla + \mathbf{A}\right)^2 \psi + \psi - |\psi|^2 \psi,$$

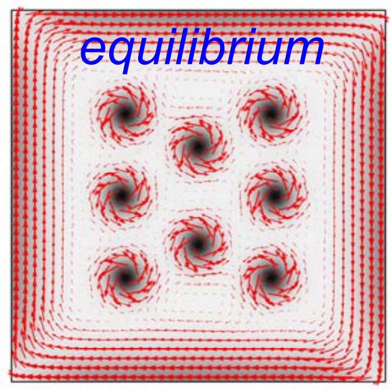
$$\sigma\partial_t\mathbf{A} = \tfrac{1}{2i\kappa}\left(\psi^*\nabla\psi - \psi\nabla\psi^*\right) - |\psi|^2 \mathbf{A} - \nabla\times\nabla\times\mathbf{A}.$$

Obtains the stationary state when $t\to\infty$, initial condition not crucial.

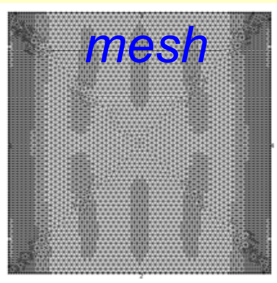
Boundary-conditions (BC):

$$\mathbf{J}^{(s)}\cdot\mathbf{n} = 0,$$

$$\nabla\times\mathbf{A} = \mathbf{B}^{(e)}.$$



Parameters:
$B^{(a)}=0.73$
$\sigma=\kappa=4, t=10^4$.



We are using the software *Comsol 4.2* [2] with time-dependent adaptive mesh refinement.

Example of time-dependent mesh for moving vortices (see below).

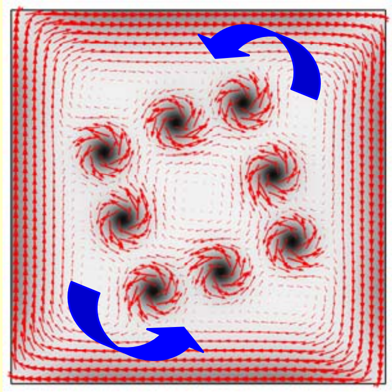
Schrödinger-Ginzburg-Landau Equation (SGLE)

It is debated if the TDGLE describe the real time evolution. It can be viewed as evolution in imaginary time, corresponding to an energy minimization of the GL functional. To model real time evolution of the Cooper pair order parameter which is conservative, we use SGLE [4]

$$-i\partial_t\psi = -\left(\tfrac{i}{\kappa}\nabla + \mathbf{A}\right)^2 \psi + \psi - |\psi|^2 \psi,$$

$$\sigma\partial_t\mathbf{A} = \tfrac{1}{2i\kappa}\left(\psi^*\nabla\psi - \psi\nabla\psi^*\right) - |\psi|^2 \mathbf{A} - \nabla\times\nabla\times\mathbf{A}.$$

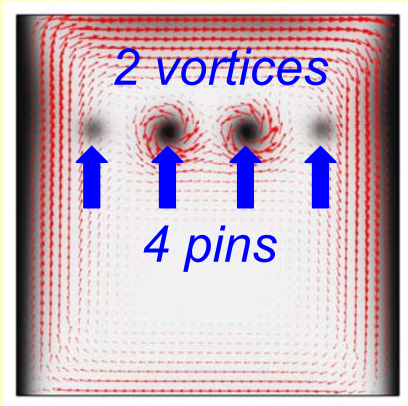
When the EM-fields are not present, the equations becomes, what is known in the context of superfluids as, the time-dependent Gross-Pitaevskii equation.



Parameters:
$B^{(a)}=0.73$, $\sigma=\kappa=4$,
$t=-225i+150$.

Pinning

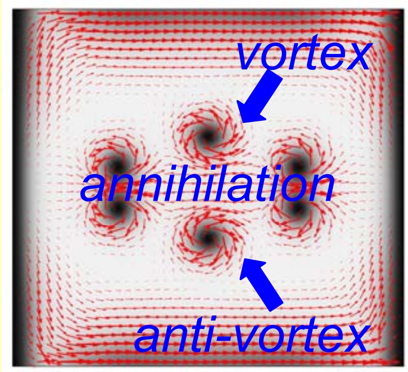
Finally we show results for the pinning of vortices with controlled impurities ("pins"). By trapping vortices the macroscopic resistivity can be decreased.
The repulsion from trapped vortices close to the surface can also prevent more vortices to enter the SC.



Parameters:
$I=0.5$, $B^{(a)}=0.5$,
$\sigma=\kappa=4, t=280$.

External Currents

To model an external-currents with TDGLE we can either change the BC for the magnetic field [3], or the BC for the normal-current where currents are injected [5].



Parameters:
$I=1.5$, $B^{(a)}=0$,
$\sigma=\kappa=4, t=160$.

Self-Consistent Boundary Conditions

Commonly used implementations of transport currents are not exact. We develop self-consistent boundary conditions that takes into account the inhomogeneity of the local currents [5].
This is important for example in the presence of vortices, when the magnetic field at the boundary is highly dependent on the local dynamics of the Cooper pairs.

References

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[3] M. Machida and H. Kaburaki, Phys. Rev. Lett. **71**, 3206 (1993).
[4] L. Peres and J. Rubinstein, Physica D **64**, 299 (1993).
[5] M. Ögren. M. P. Sørensen and N. F. Pedersen, conference proceedings.